NUMERICAL WEATHER PREDICTION AND CHAOS

Predictability

Credentials

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Outlook

- **Numerical Weather Prediction (NWP)**
  - **Microscopic and macroscopic approaches**
  - **Dynamical basis: conservation laws**
  - **Parameterizations**
  - **Numerical methods and grids**

- **Predictability**
  - **Atmospheric Chaos: initial conditions sensitivity**
  - **Intrinsic uncertainties of NWP models**

Basic concepts

- The numerical weather prediction is currently based on the application of the classic physic laws, that is Newton dynamics, thermodynamics and the hydrodynamics laws of the atmospheric fluid, in order to simulate its future evolution from the observation of its current state.
- Nowadays, NWP models are the essential tool on the elaboration process of meteorological forecasts.
- The evolution and the application of NWP models have walked hand in hand with the evolution of informatic hardware.

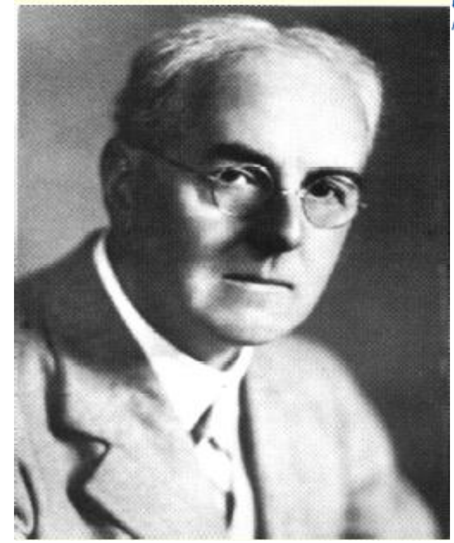
History

- The weather forecast is a mathematical problem of initial values !!!

1904: Vilhelm K.F. Bjerknes theorized that it could be possible to forecast the weather if there were enough precise knowledge of:

1. Initial state of the atmosphere; and
2. The physical laws which describe the atmospheric evolution from one state to another.

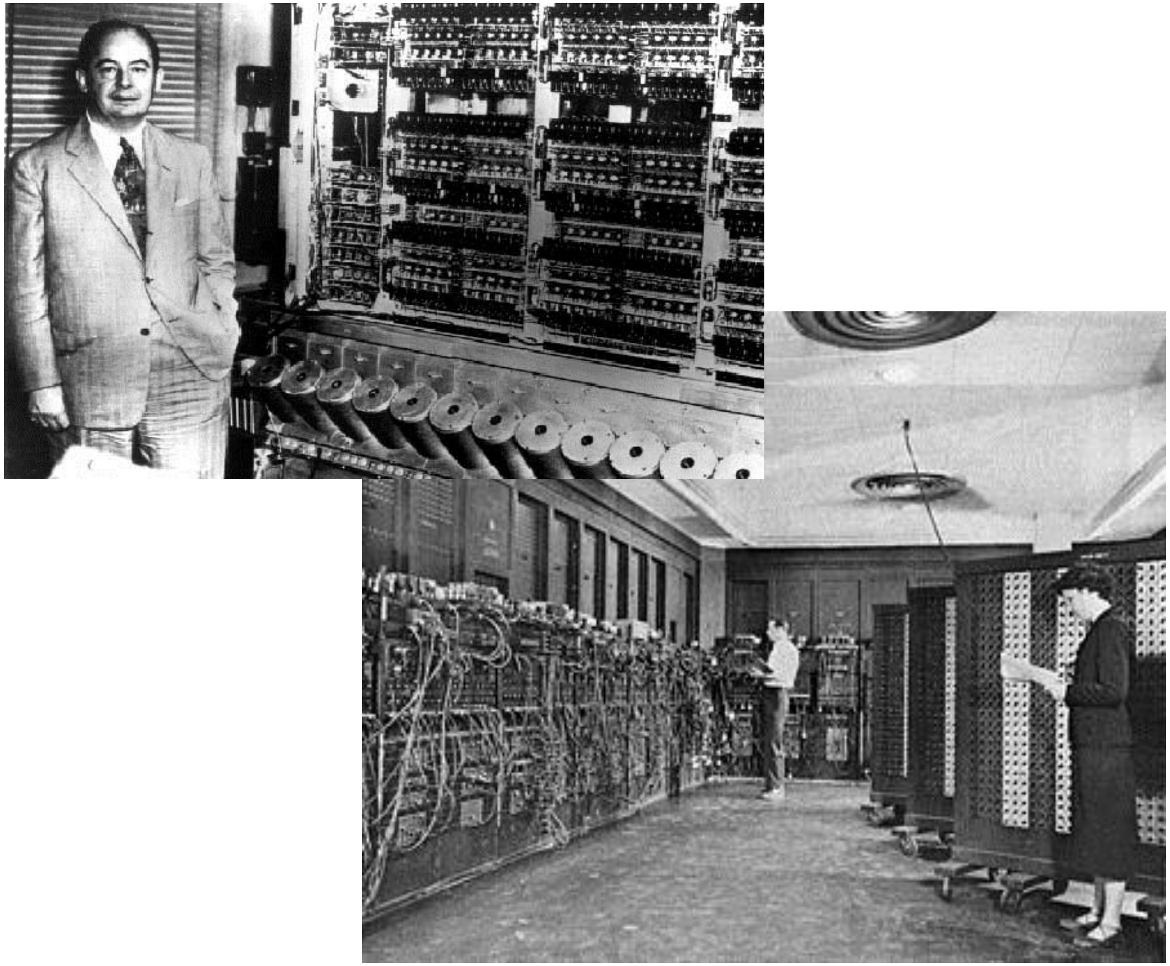
History



- His attempt did not work: the problem, the sound waves !!!

1922: Lewis Fry Richardson carried out the first numerical weather prediction dealing with the mathematical problem using the new finite increments methodology.

History



1950: the first successful numerical weather prediction never carried out was done by a meteorologist team (Charney, Thompson, Gates, Fjörtoft and von Neumann) using ENIAC super-computer.

- **It was used a simplified form of atmospheric dynamics based on the barotropic vorticity equation.**

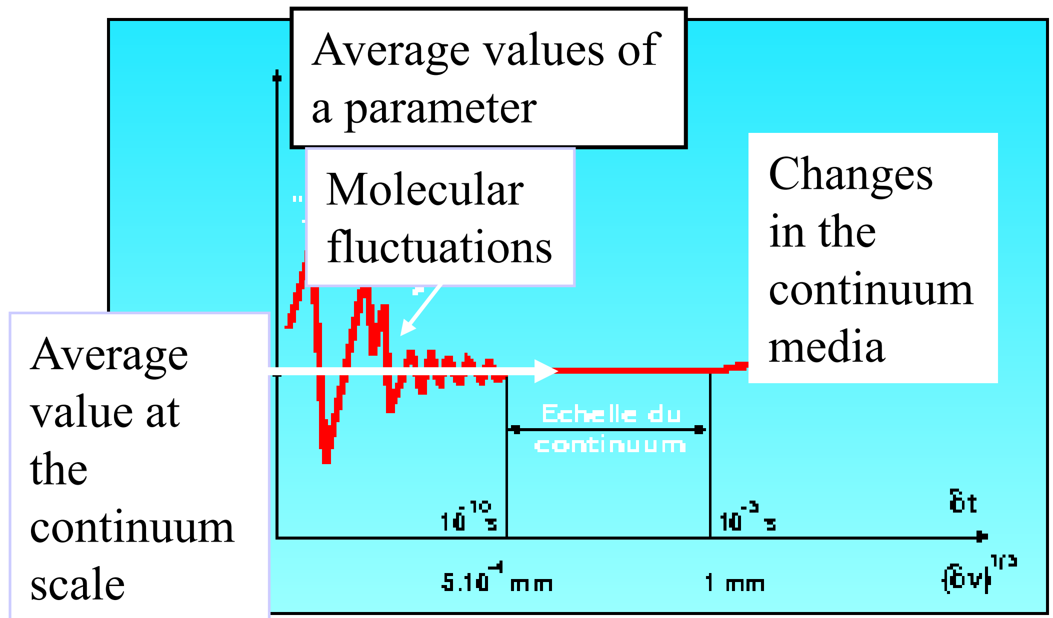
Microscopic point of view

- A first possible physic approximation to the numerical weather prediction could be done simulating each atmospheric molecule evolution and taking into account the collisions between them.
- Actually one has a huge number of ordinary equations in principle easy to solve.
- But we have not the enough calculation power (in spite of using super-computers), neither the possibility to know accurately the initial state conditions due to the Uncertainty Principle of Heisenberg (i.e. when one tries to observe the state, one changes it !!!).

Macroscopic point of view

- Thermodynamics and fluid mechanics provides us with a second approach: the macroscopic one.
- It hypothesis that at some big enough scales we could consider the atmospheric fluid as a continuum *medium*.

It is not necessary anymore to know the exact position of each atmospheric molecule, instead we will have enough with macroscopic variables easier to measure like **temperature, pressure, wind and humidity.**



Fundamental conservation laws

The application to the atmospheric fluid of the fundamental conservations laws of Newton classic Physics !!!

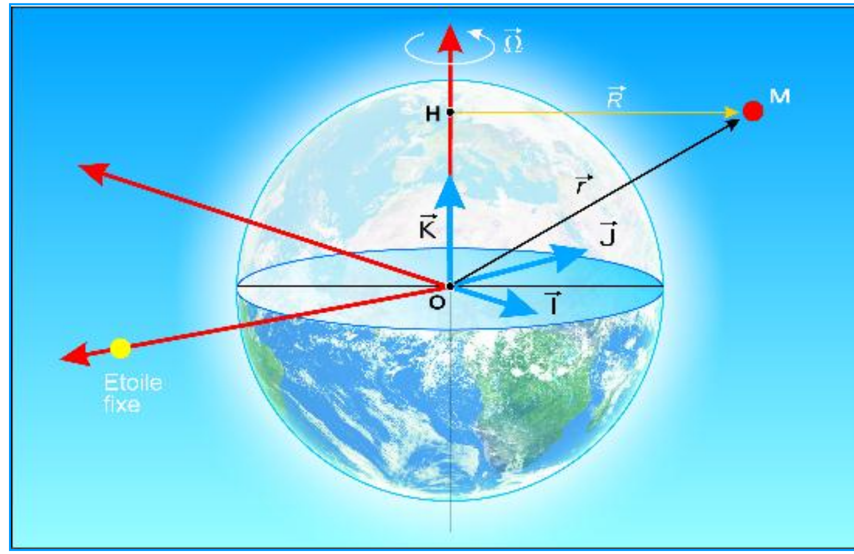
- **linear momentum conservation in a rotating system (linear momentum equations);**
- **Energy conservation (thermodynamic energy equation); and**
- **Mass conservation (continuity equation).**

+ Ideal gas law

Linear momentum conservation: 2nd law of Newton

$$\frac{D\boldsymbol{V}_a}{Dt} = \boldsymbol{F}$$

Newton 2nd law



Forces which apply to the atmospheric fluid

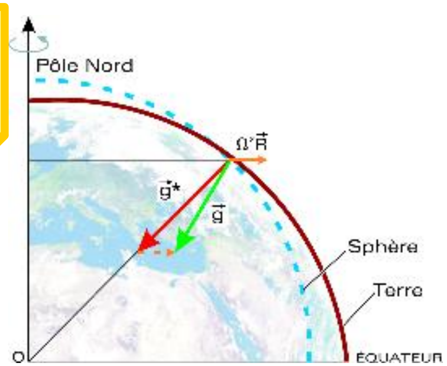
$$\frac{d\boldsymbol{v}}{dt} = \boxed{-2\boldsymbol{\Omega}\times\boldsymbol{v}}\boxed{-\alpha\nabla p}\boxed{-g\boldsymbol{k}}\boxed{+\boldsymbol{\tau}}$$

Friction forces

Apparent Coriolis force

Pressure gradient force

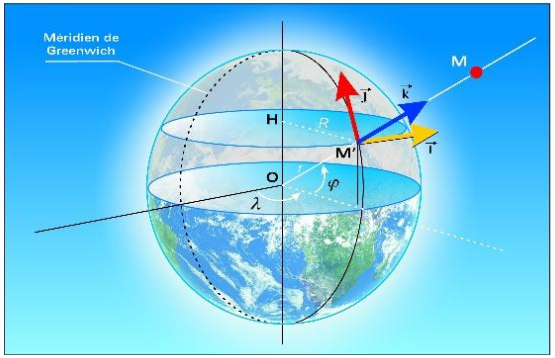
Gravity force



linear momentum conservation: the three components

$$\frac{du}{dt} = 2\Omega v \sin\theta - 2\Omega w \cos\theta + \frac{uv}{r}\tan\theta - \frac{uw}{r} - \frac{1}{\rho r\cos\theta}\frac{\partial p}{\partial \lambda} + \tau_\lambda$$

$$\frac{du}{dt} = 2\Omega v \sin\theta + \frac{uv}{a}\tan\theta - \frac{1}{\rho a\cos\theta}\frac{\partial p}{\partial \lambda} + \tau_\lambda$$



$$\frac{dv}{dt} = -2\Omega u \sin\theta - \frac{u^2}{r}\tan\theta - \frac{vw}{r} - \frac{1}{\rho r}\frac{\partial p}{\partial \theta} + \tau_\theta$$

$$\frac{dv}{dt} = -2\Omega u \sin\theta - \frac{u^2}{a}\tan\theta - \frac{1}{\rho a}\frac{\partial p}{\partial \theta} + \tau_\theta$$

Analysis scale simplifications !

$$\frac{dw}{dt} = 2\Omega u \cos\theta + \frac{u^2+v^2}{r} - \frac{1}{\rho}\frac{\partial p}{\partial r} - g + \tau_r$$

$$\frac{dw}{dt} = -\frac{1}{\rho}\frac{\partial p}{\partial z} - g + \tau_r$$

Another simplifications:

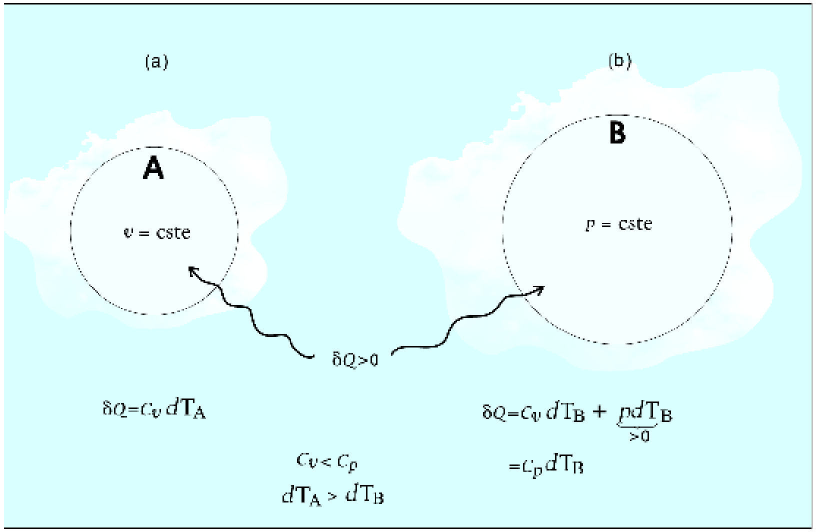
$r = a + z \approx a$

$g(z) \approx g_o$

Energy conservation: First law of thermodynamics

$$\Rightarrow c_v \frac{dT}{dt} = Q - p \frac{d\alpha}{dt}$$

$$c_p \frac{dT}{dt} = Q + \frac{RT}{p}\omega$$

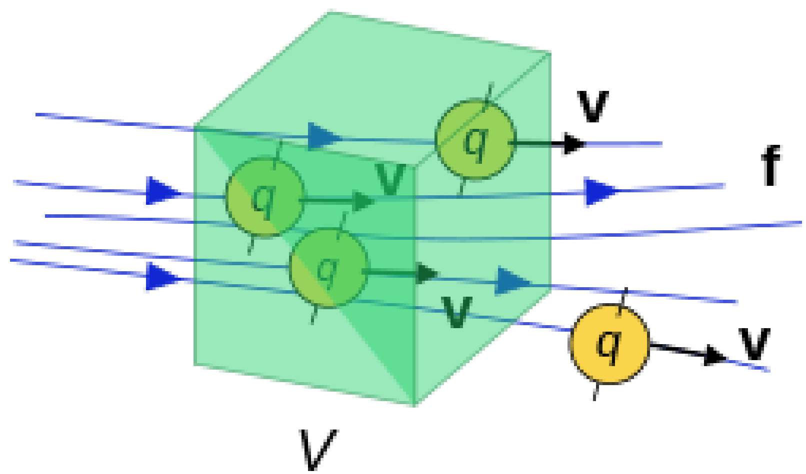


Mass conservation: continuity equation

$$\Rightarrow dM = 0$$

$$\Rightarrow \frac{d}{dt}\ln\rho = -\nabla\cdot\boldsymbol{v}$$

$$\Rightarrow \frac{\partial\rho}{\partial t} = -\nabla\cdot(\rho\boldsymbol{v})$$



$\mathbf{f} = \varphi\mathbf{v}$

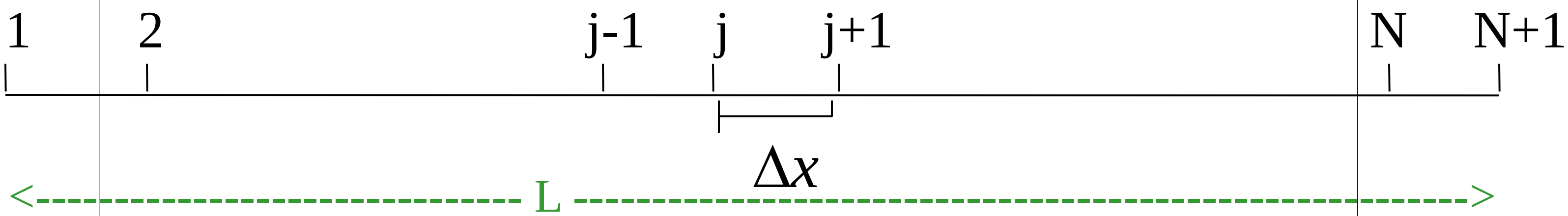
$\varphi = dq/dV$

Primitive equations

- **Unluckily: the resulting system of equations is highly non-linear and it has not analytical solution.**
- **Luckily: mathematics provides us with a tools to solve them approximately: the numerical methods.**

Discretizing in space and time, these numerical methods allows us to obtain an ordinary system of equations (like the initial case of molecular approximation !!!).

Numerical methods: finite differences



$$\phi(x) \Rightarrow \phi_j \quad (j = 1, \ldots, N)$$

Taylor series expansion:

$$\phi_{j+1} = \phi_j + \phi'_j \Delta x + \frac{1}{2}\phi''_j (\Delta x)^2 + \frac{1}{3!}\phi'''_j (\Delta x)^3 + \ldots$$

$$\phi_{j-1} = \phi_j - \phi'_j \Delta x + \frac{1}{2}\phi''_j (\Delta x)^2 - \frac{1}{3!}\phi'''_j (\Delta x)^3 + \ldots$$

Numerical methods: three schemes of finite differences

$$\frac{\phi_{j+1}-\phi_j}{\Delta x} = \phi'_j + \frac{1}{2}\phi''_j\Delta x + \frac{1}{3!}\phi'''_j\Delta x^2 + O(\Delta x^3)$$

forward approximation   Consistent if ϕ''_j, ϕ'''_j, ... are bounded

$$\frac{\phi_j-\phi_{j-1}}{\Delta x} = \phi'_j - \frac{1}{2}\phi''_j\Delta x + \frac{1}{3!}\phi'''_j\Delta x^2 + O(\Delta x^3)$$

backward approximation

Combination of both previous:

$$\frac{\phi_{j+1}-\phi_{j-1}}{2\Delta x} = \phi'_j + \frac{1}{3!}\phi'''_j\Delta x^2 + O(\Delta x^4)$$

centred differences   Consistent if ... are bounded

numerical methods properties

Convergence

Discrete solution ---------> continuum solution

Discretization each time smaller

Consistency

Discretized equation ---------> continuum equation

Stability

Discrete bounded solution

Lax-Richtmeyer theorem

If a discretizating scheme is consistent and stable, then it has to be convergent, and vice versa.

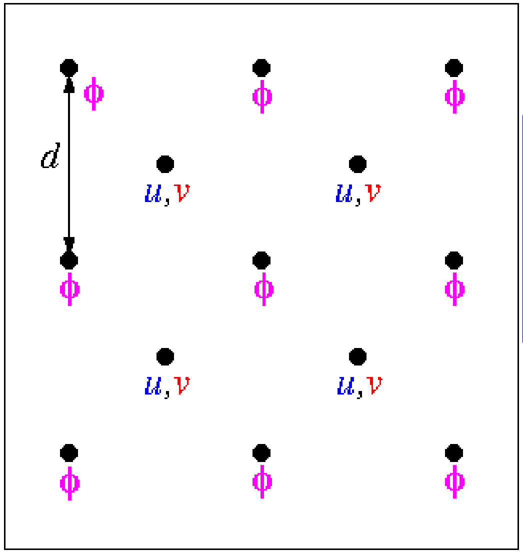
CFL condition:
Courant-Friedrichs-Lewy

$$\boxed{\frac{u \cdot \Delta t}{\Delta x} < C}$$

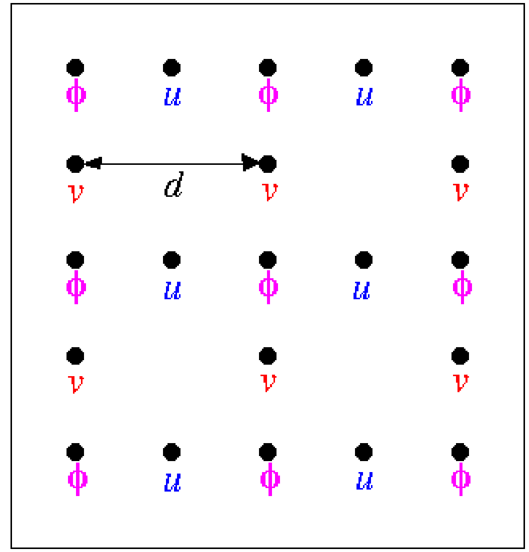
We cannot use an Δt if the maximum NWP model advection $u \cdot \Delta t$ due to u velocity goes further than the current Δx grid discretization distance.

- **If the CFL condition is violated the NWP model *blows up* and its results lose any meteorological coherence.**

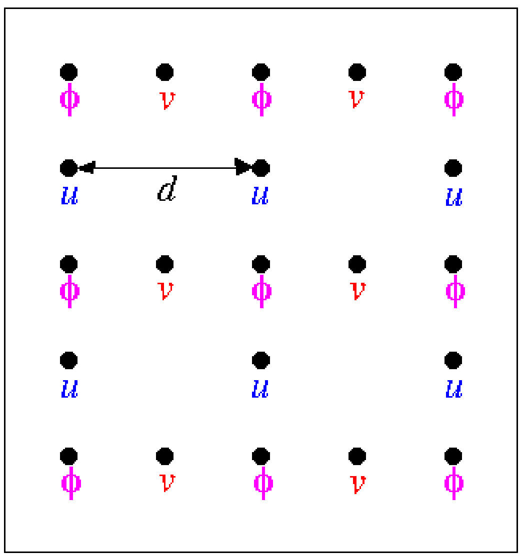
Grids or *webs*: types



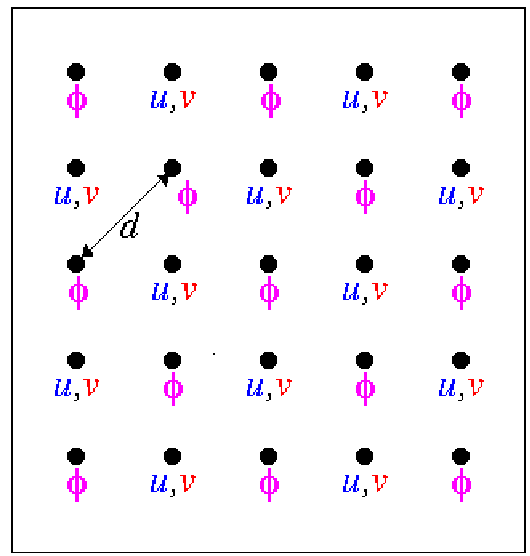
Grid B (UM)



Grid C (HIRLAM, DWD, ECMWF)

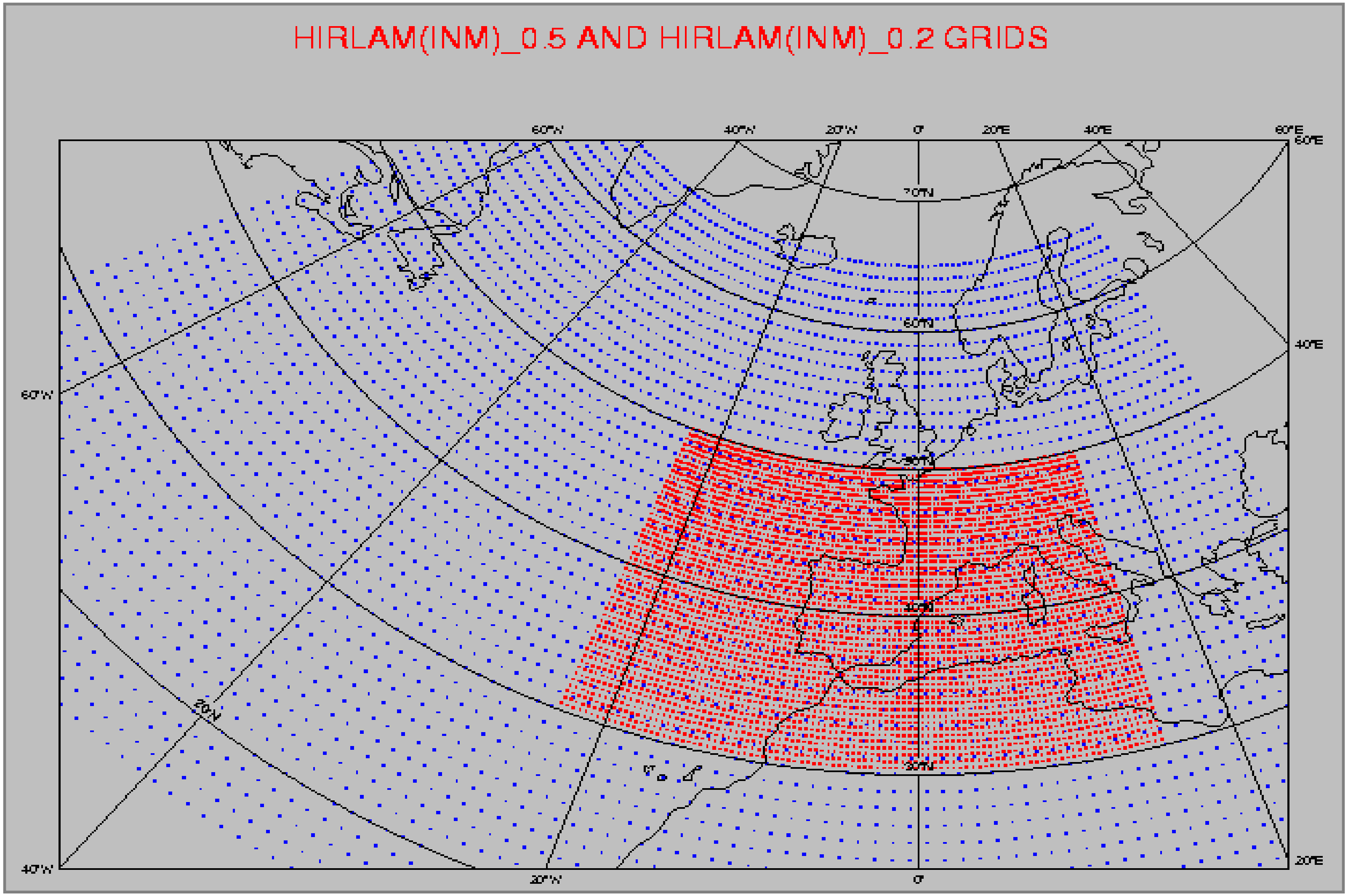


Grid D (DNMI)



Grid E (ETA)

Grids or *webs*



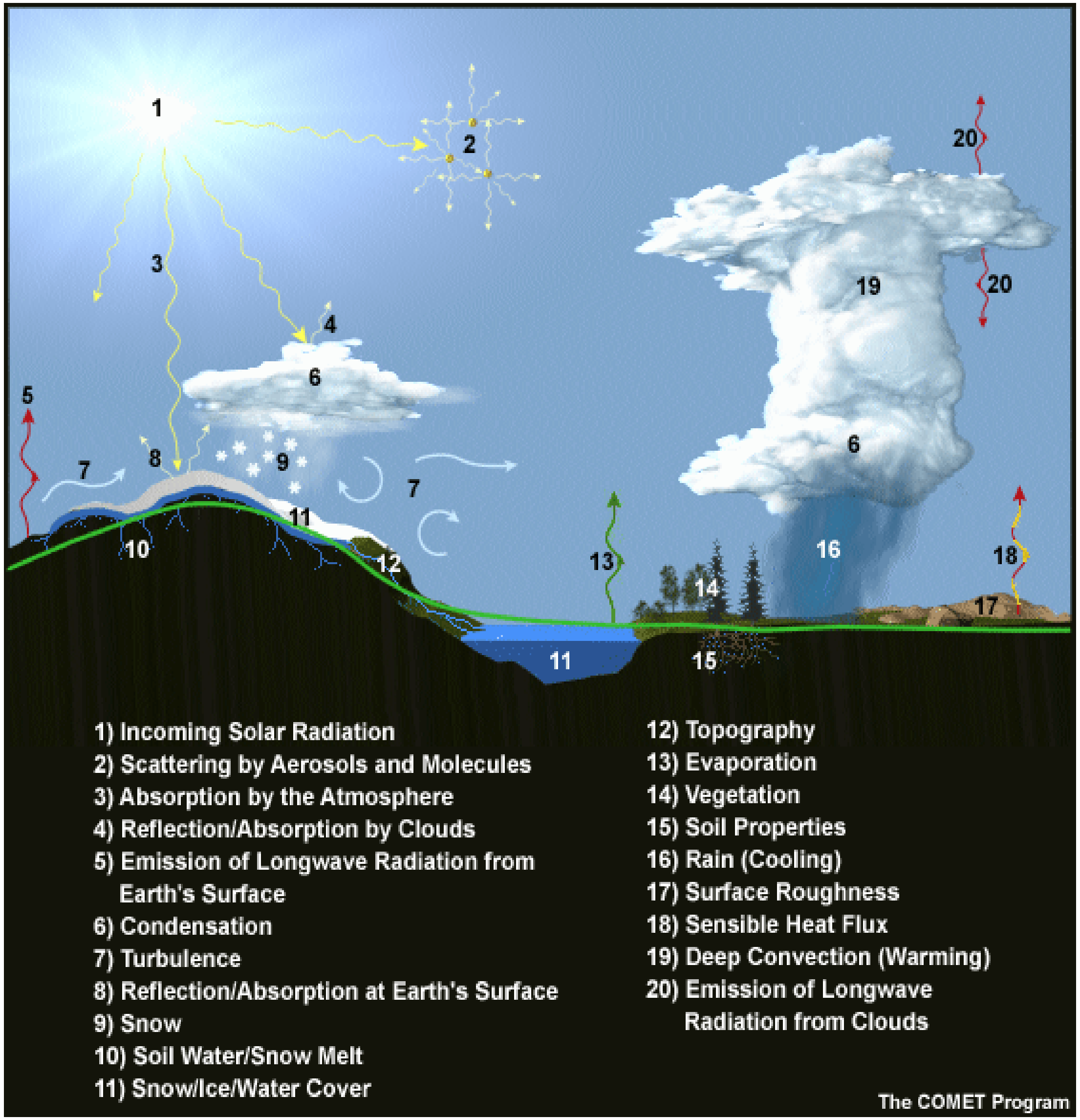
Parameterizations

- There are processes which they are not solved by the discretized equations:
  - Scales smaller than the solved one (sub-grid process): convection.
  - Different physics processes: cloud microphysics.
  - Complex and expensive time computer processes to be included directly in NWP models: radiation.
  - Boundary conditions not enough described: orography.

Parameterization hypothesis: there is an statistic ensemble of sub-grid processes with secular and local equilibrium with the grid resolved processes.

- **Atmospheric fluid equations are multi-scale ones:**
  - **NWP models become quickly wrong when the average influence of sub-grid process are not taking into account in resolved scales.**

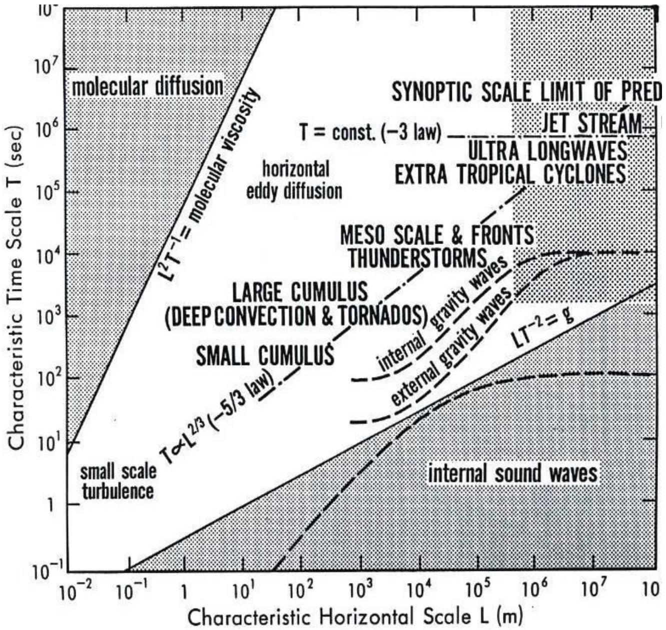
Parameterized processes

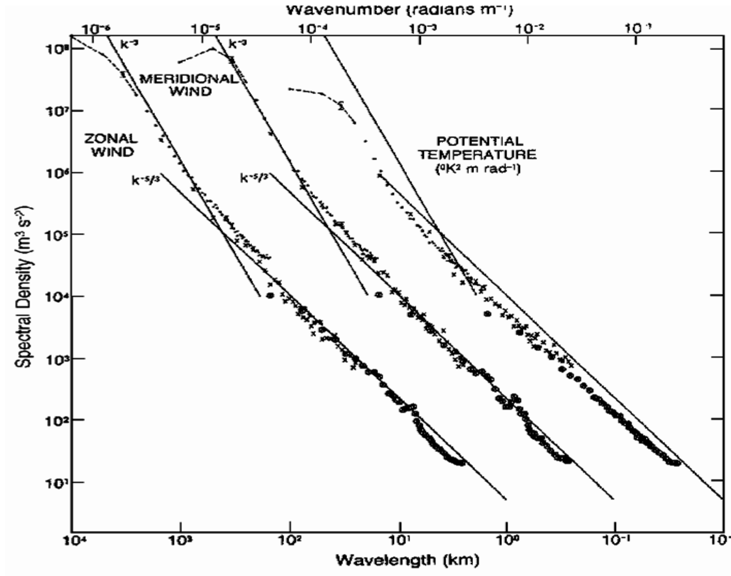


The parameterit-zacions of unresolved sub-grid processes has had and has a very positive impact improving the NWP model forecasts.

The **atmospheric mineral dust** could be one of the parameteri-zations !!!

Parameterizations: structural problems



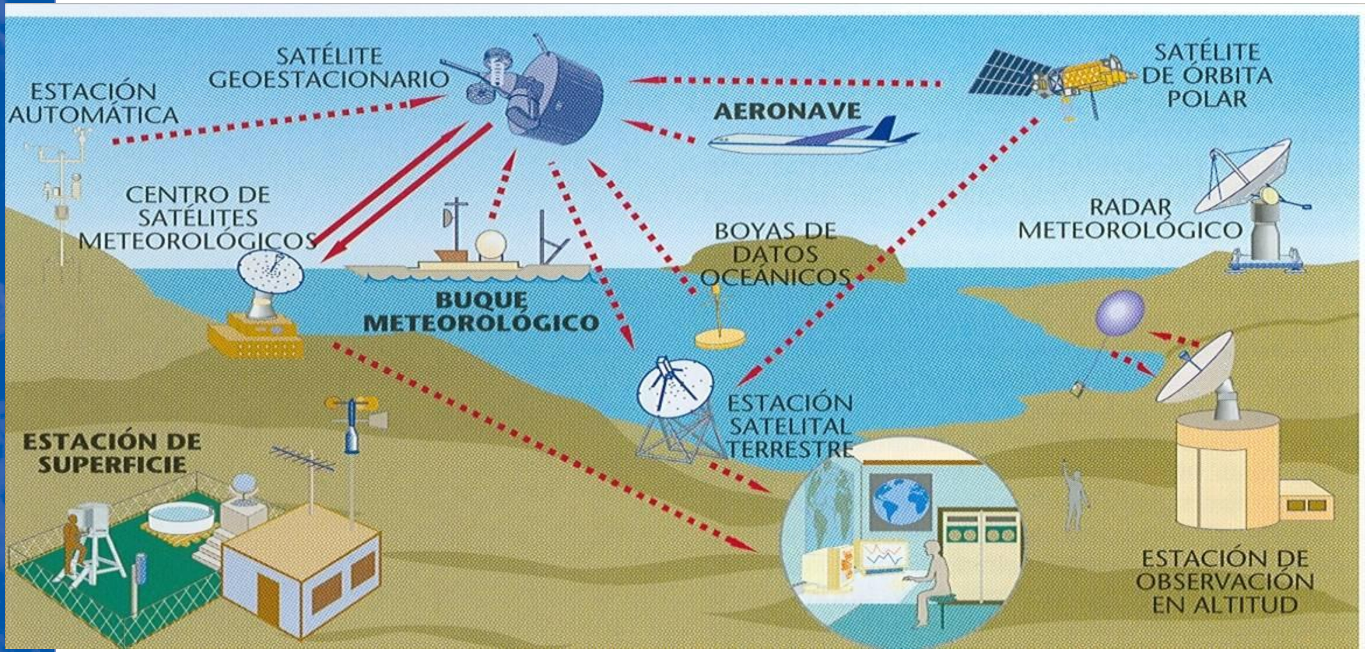


Alternative hypothesis: there is not a secular equilibrium between the NWP model unresolved scales with the resolved ones: **there is not an spatio-temporal spectral gap between them in the observed atmosphere.**

- **And neither there is a local equilibrium in the grid points:**
  - **Where the grid model processes are poorer and worse resolved, that is in each individual grid point, is where the parameterizations are.**

Chaos

- The system of highly non linear differential partial equations used in the numerical weather forecast has a high sensitive dependence on the small uncertainties of the initial conditions (observations). In other words, the atmospheric simulation is a chaotic dynamical system.

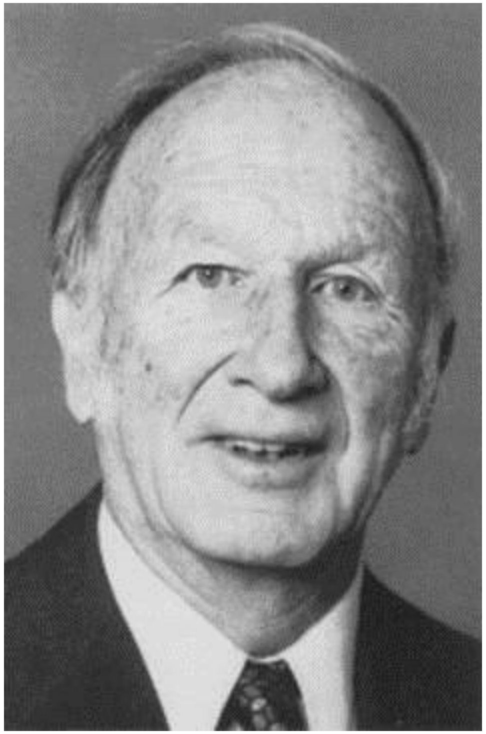


Pre-process: assimilation of the observations in order to set up an initial atmospheric state to run a NWP model.

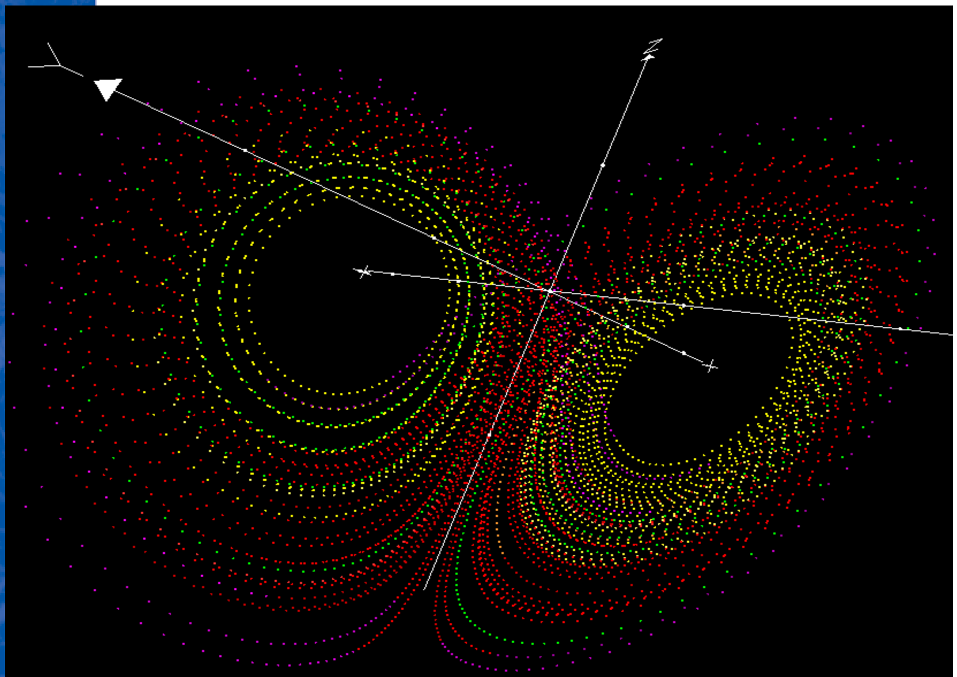
Sensitive dependence on initial conditions

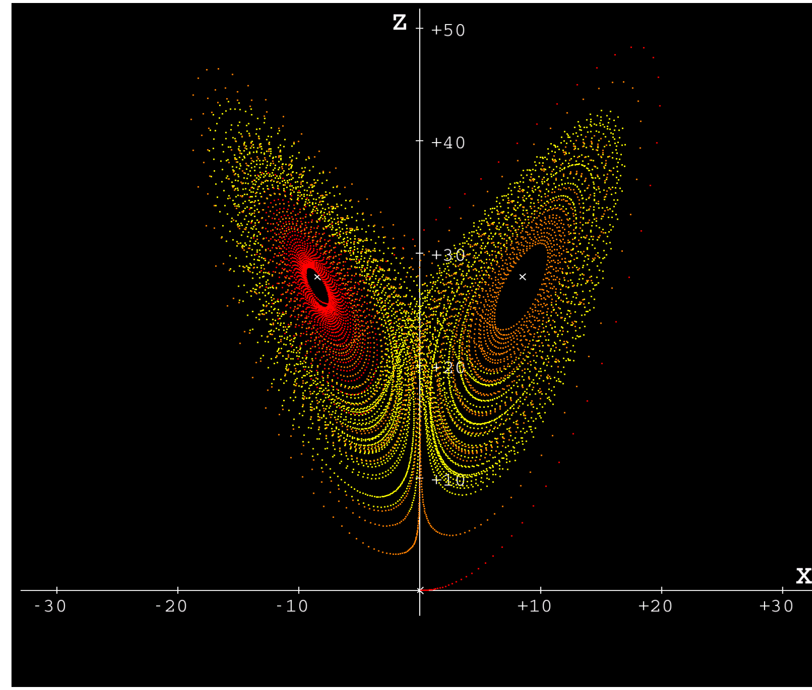
- **Edward Lorenz (1917-2008) found out that it will be never possible to do a *perfect* numerical weather prediction *(Lorenz E., 1963)*.**

And he explained the sensitive dependence on initial conditions in a metaphorical way with the well-known ***butterfly effect***, saying: “Does the flap of a butterfly’s wings in Brazil set off a tornado in Texas?”; The phrase refers to the idea that a butterfly’s wings might create tiny changes in the atmosphere that may ultimately alter the path of a tornado or delay, accelerate o even prevent the occurrence of a tornado in another location.



1963 Lorenz model





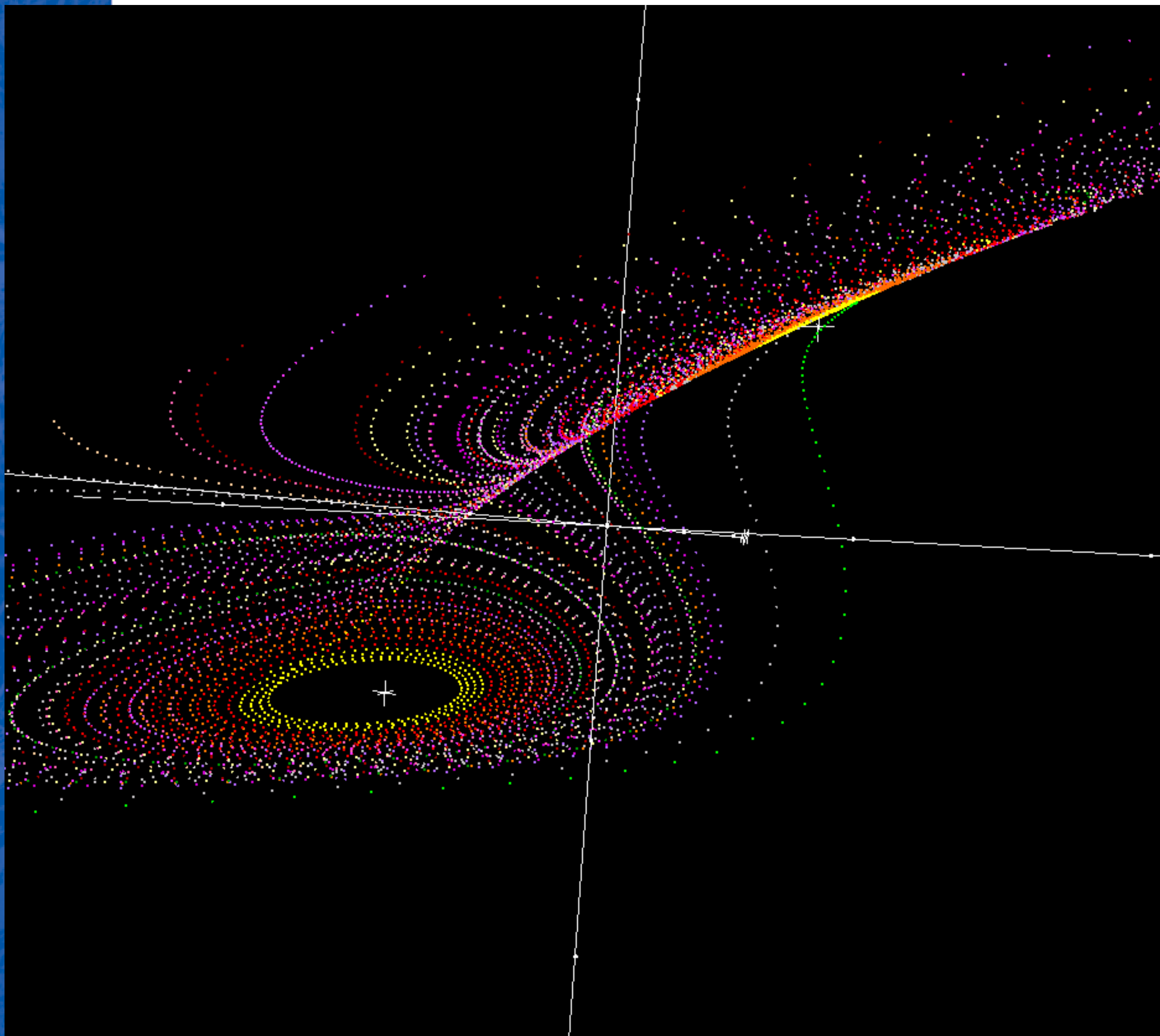
Lorenz run in “primitive” computer, the Royal McBee, a quite simplified atmospheric equations, but keeping the essence of them, specifically a system of non linear differential equations derived from an *intensive* truncation of a spectral thermal convection model.

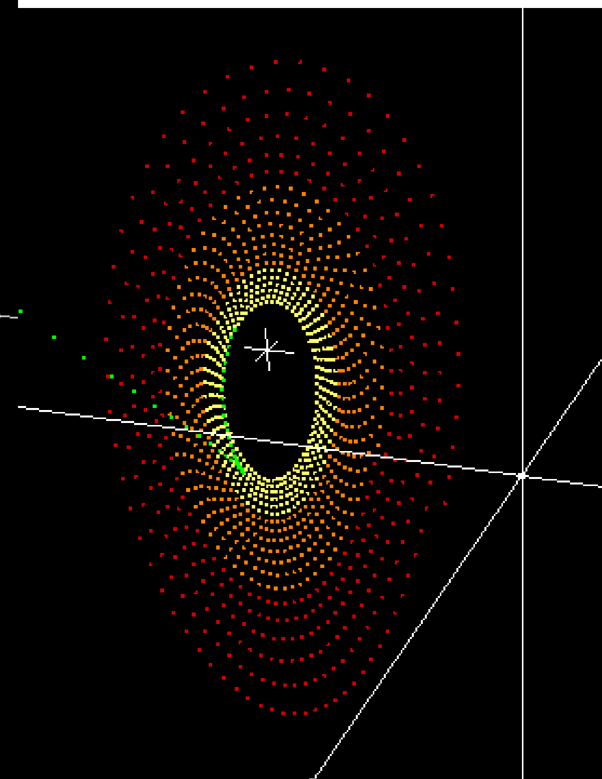
$$\frac{dx}{dt} = \sigma(y - x)$$

$$\frac{dy}{dt} = x(\rho - z) - y$$

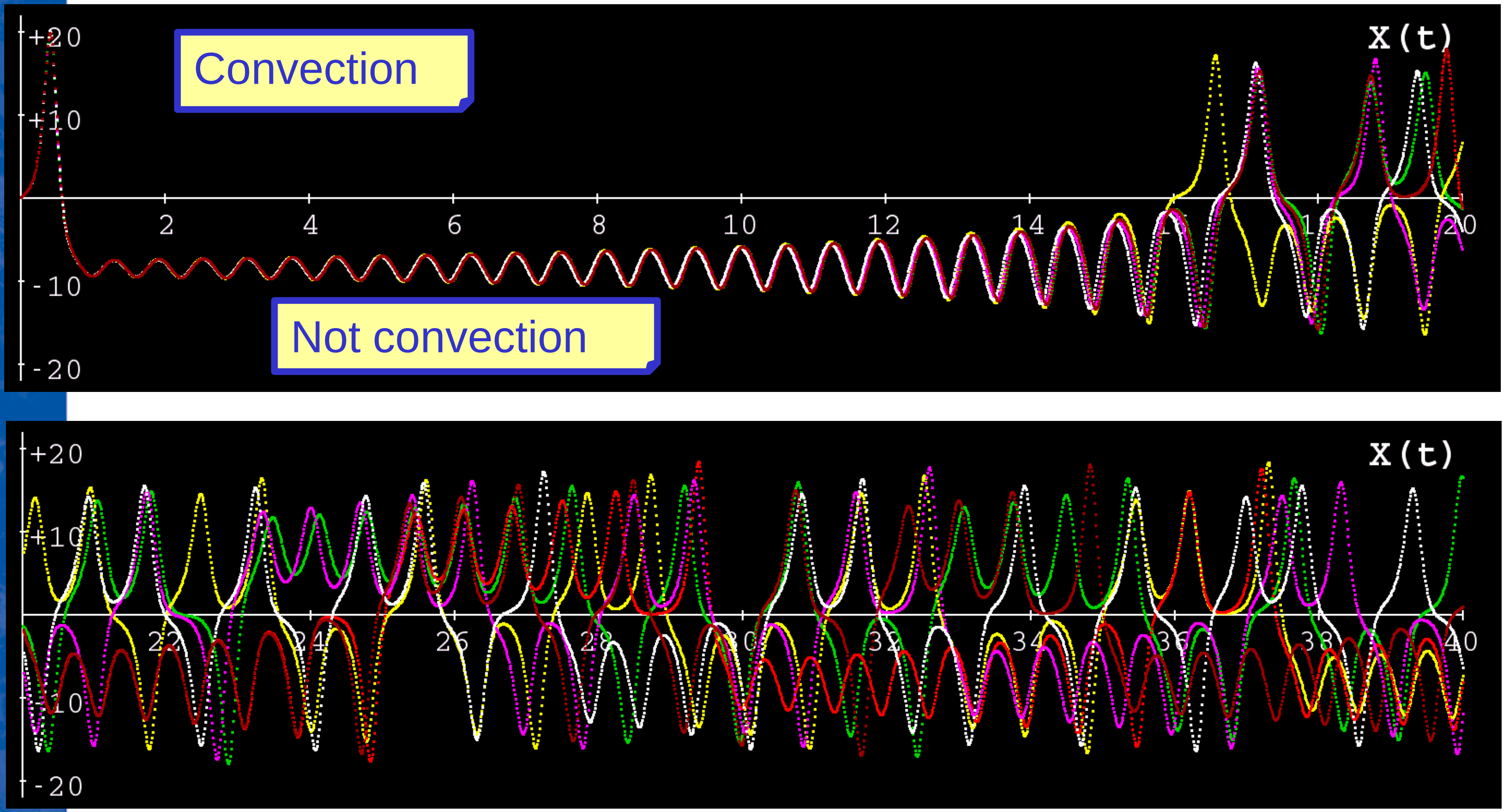
$$\frac{dz}{dt} = xy - \beta z$$

1963 Lorenz model





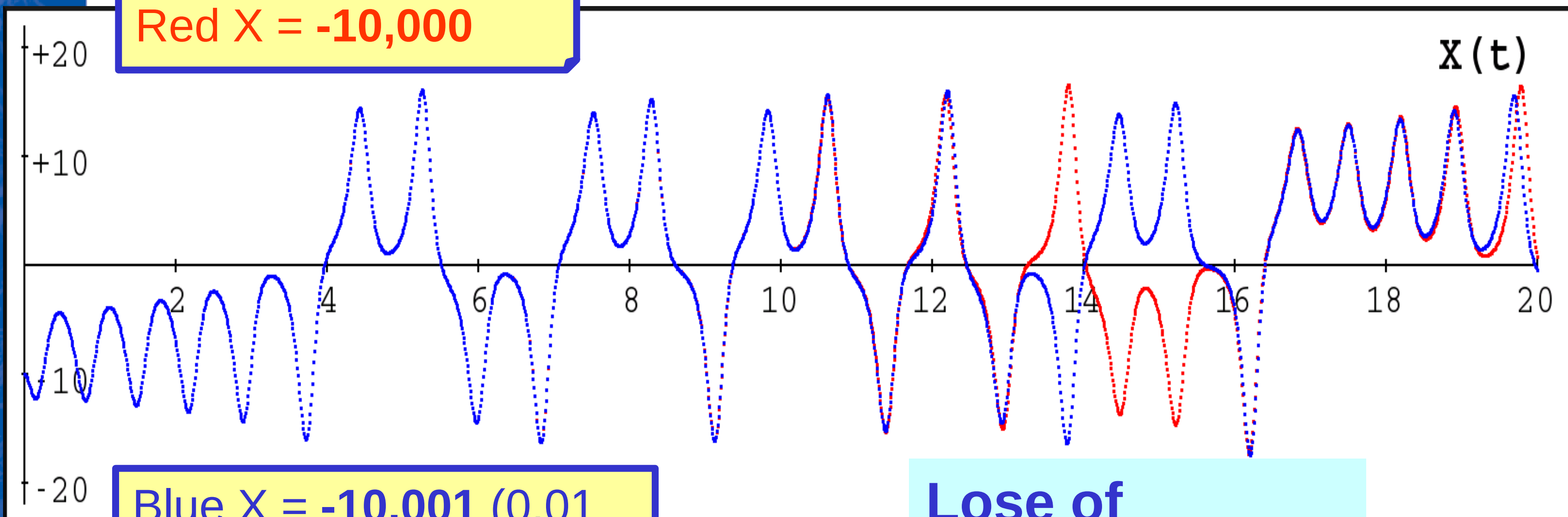
1963 Lorenz model



Sensitive dependence on the initial conditions

Lorenz realized himself, and this was really extraordinary at that time, that an insignificant difference in the initial values could lead with enough time to a divergent forecast.

Red X = **-10,000**



Blue X = **-10,001** (0,01 % of variation)

Lose of predictability

Consequences of the sensitive dependence on the initial conditions

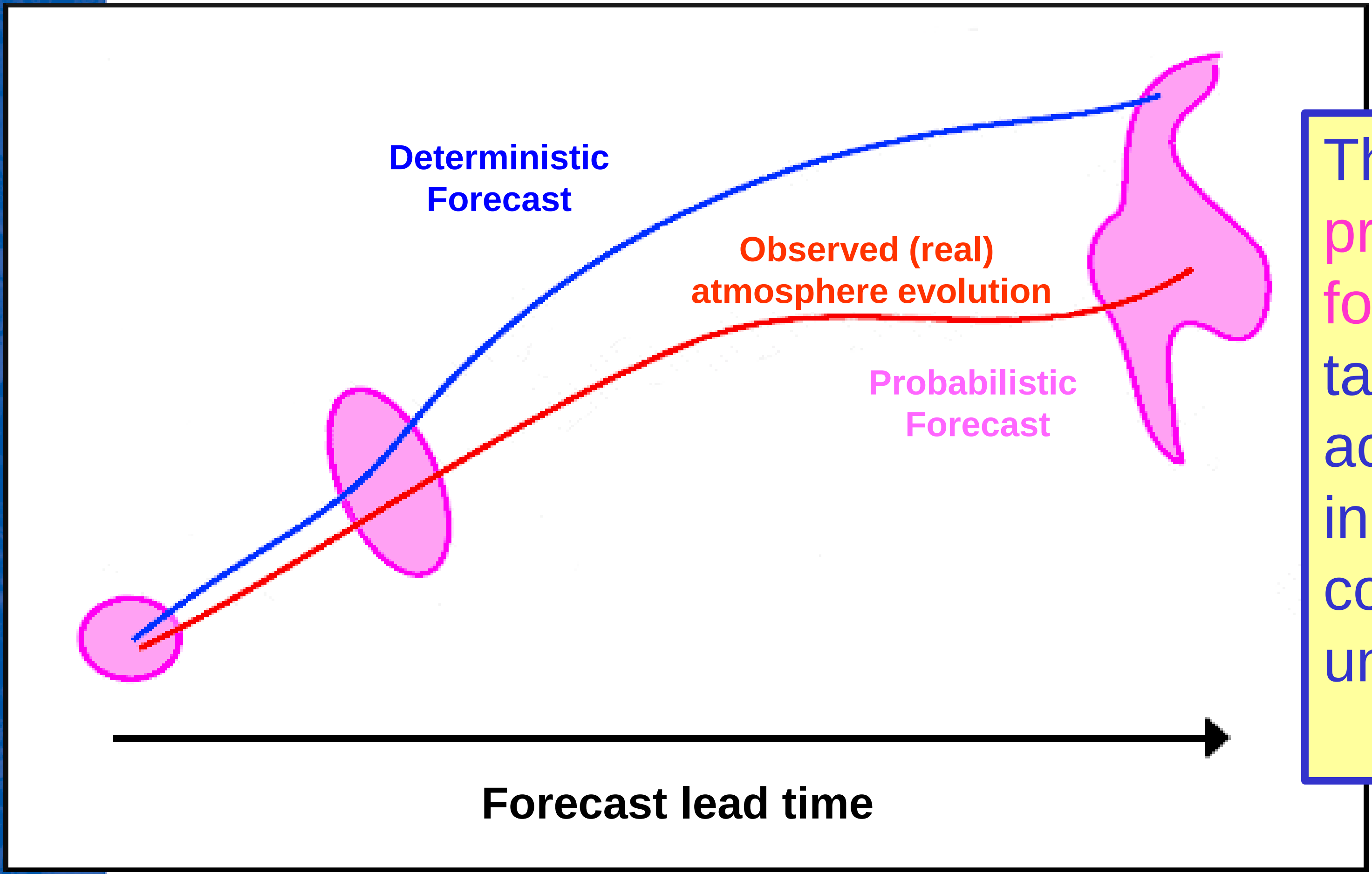
- This problem has an *impossible solution*:
  - It is needed exact (~perfect) meteorological observations, that is infinitely precise: their values would have to have an infinite decimal numbers.
  - The super-computers would have to work with an infinite decimal number positions.
- But both of previous points will be never possible.
- On consequence, NEVER will be possible to do a ***PERFECT weather forecast***.

Lorenz thinking way !!!

Ensembles weather forecast

- Somehow if it is possible to know the uncertainty of the observations (through Probability Distribution Functions or PDFs), it will be possible to forecast the more probable future state of the atmosphere.
- The theoretical application, the Fokker-Planck equation (based on Liouville equations), could not be properly solved for the atmospheric case.
- The practical feasible solution is to integrate (run) several NWP deterministic models with a slightly different initial conditions, but all of them equally possible and compatible with the underlying uncertain observations:
  - We call this approximation an *ensemble forecast*.
- The forecast change their deterministic character to a probabilistic one.

Theory of the probabilistic forecast

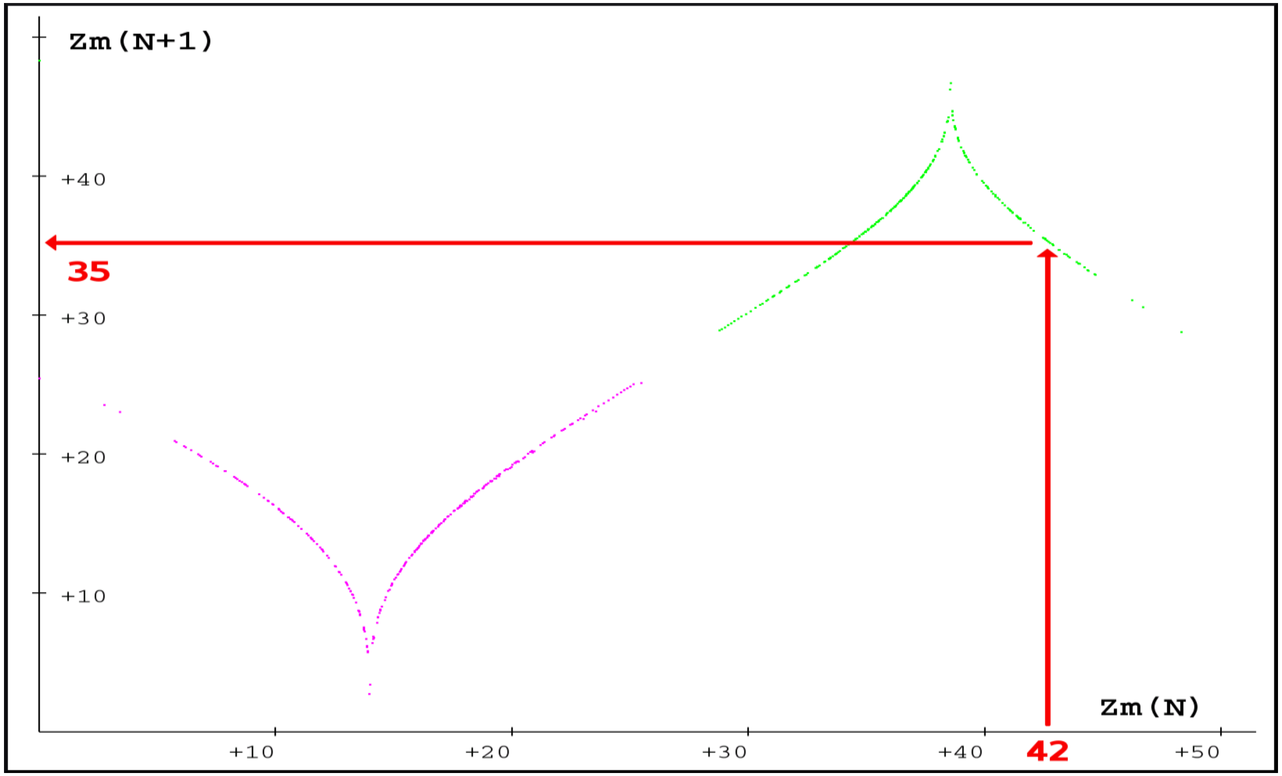


The probabilistic forecast takes into account the initial condition uncertainties.

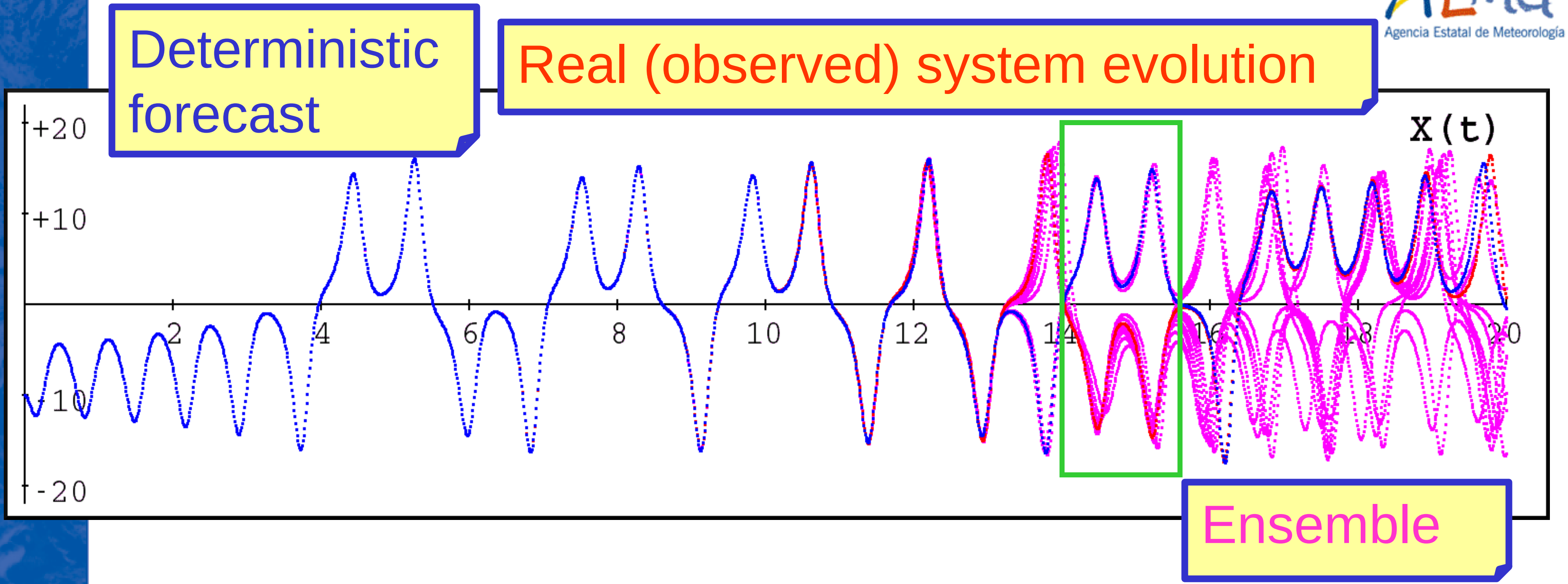
The PDF area is a predictability measure: the smaller (bigger) PDF area the higher (lower) is the predictability.

Predictability limits

- Short range (up to 3 days): α-mesoscale
- Medium range (from 3 to 15 days): synoptic scale
- Monthly (from 15 to 30 days): ¿planetary waves?
- Seasonal (from 1 month to 1 year): e.g. Tropics

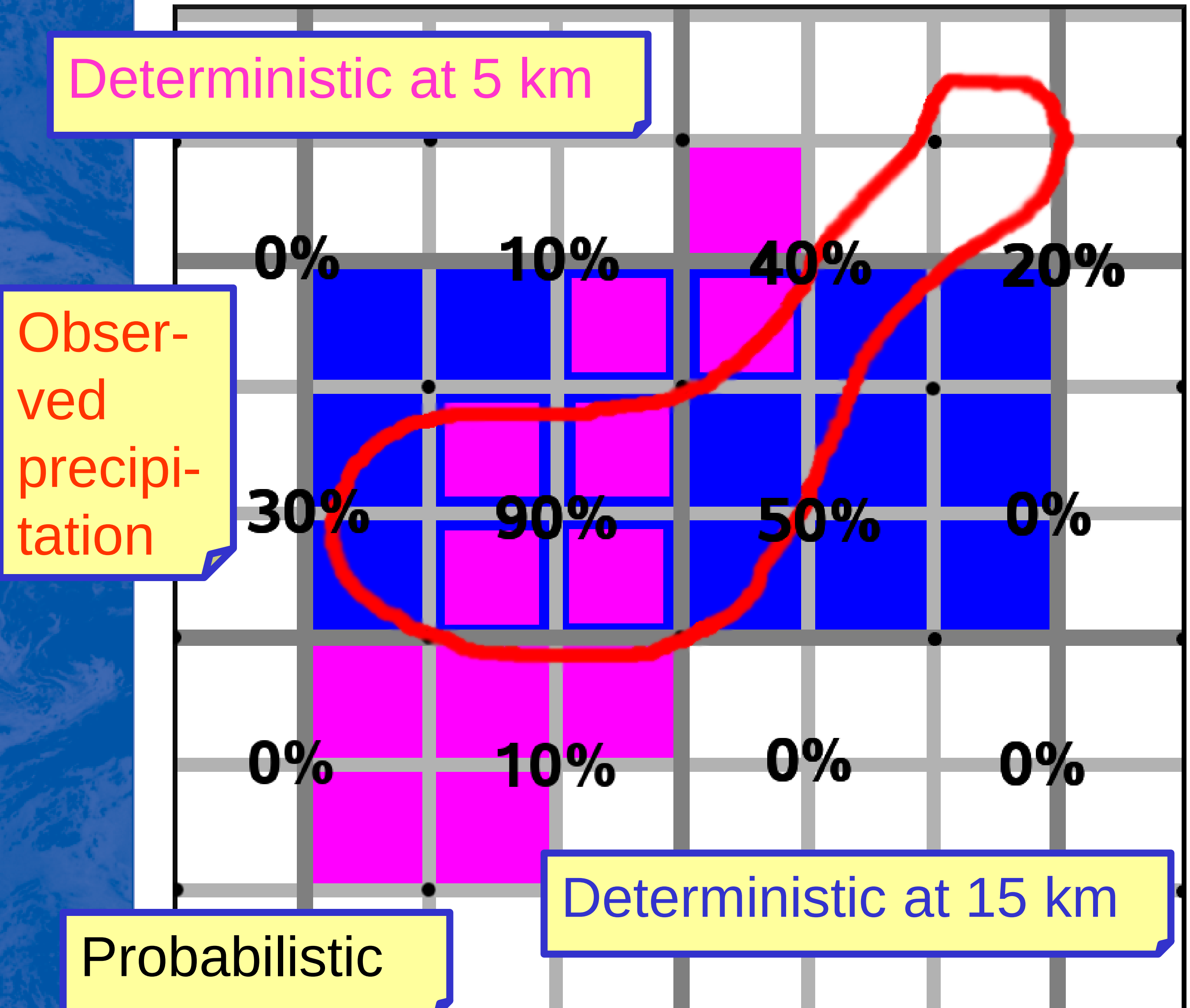


Probabilistic forecast exercise



In the temporal section from 14 to 16 with an erroneous deterministic forecast, the probabilistic forecast will be that there is a 20% of chances (2 members of 10) of convective thunderstorms and a 80% of probabilities (8 members of 10) without them, quite close to the real system evolution without convection. It has to be noted that taking into account the initial condition uncertainties we have been able to improve deterministic forecast of no convection (0% of chances) with the probabilistic one (80% of chances).

Probabilistic forecast example



The precipitation probabilistic forecast has the best verification !!!

Example of the initial condition uncertainties and NWP model errors.

Example of an AEMET-SREPS real case.

Main messages to remember

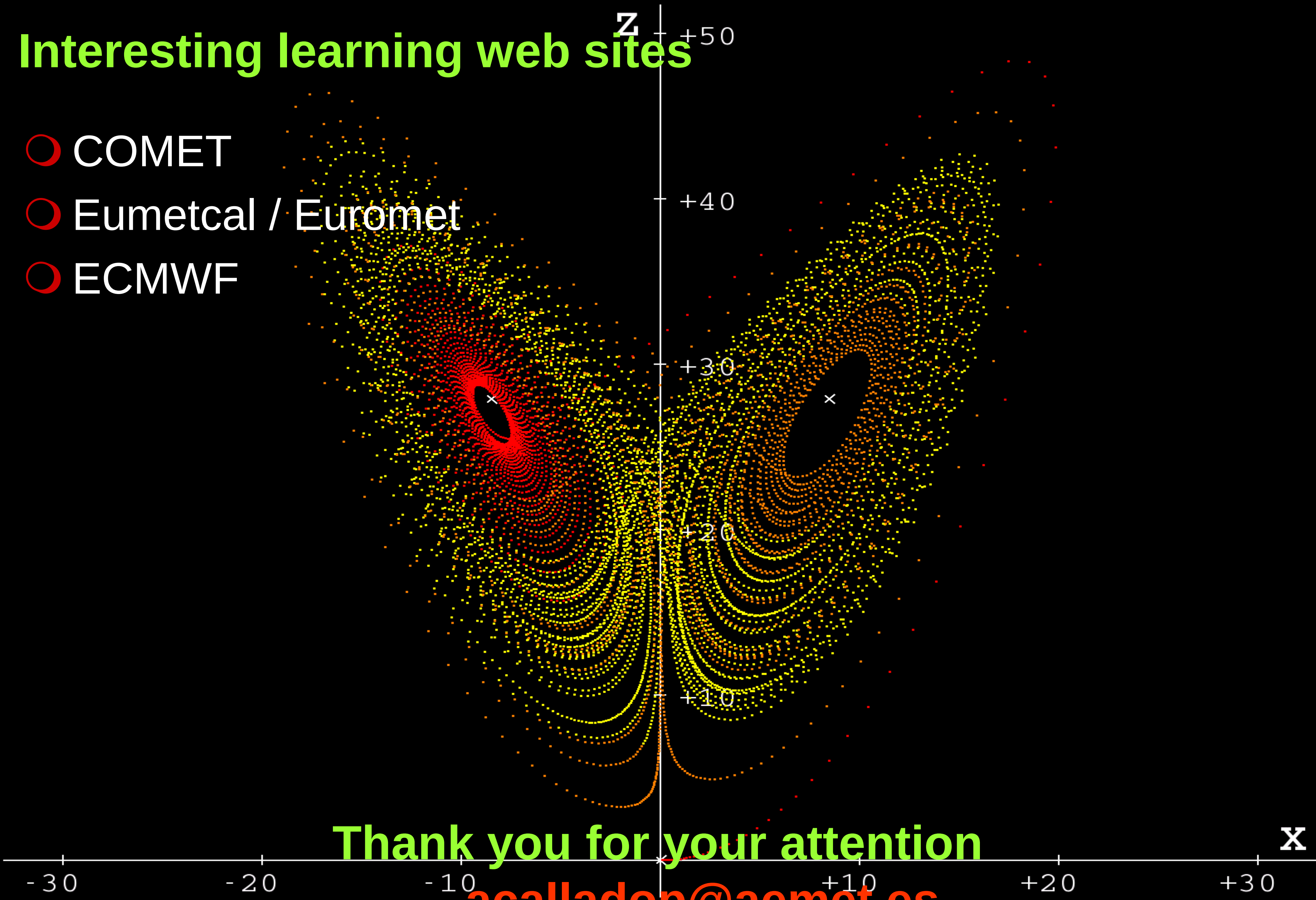
- A **Numerical Weather Prediction model** is a **SIMULATION** of the evolution of the atmospheric states: that is, it is an ***informatic program*** which has most of our current knowledge about the atmosphere.

- It will **NEVER** be possible to do a **PERFECT FORECAST** of the future state of the atmosphere.

- But it is possible to estimate the **predictability** of the future atmospheric states and limit their uncertainty through **probabilistic forecasts** drawn from ***ensembles***.

Interesting learning web sites

- COMET
- Eumetcal / Euromet
- ECMWF



Thank you for your attention

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